International Journal of Theoretical Physics, Vol. 37, No. 9, 1998

String Cosmology with Brans–Dicke Theory in Higher Dimensional Space-Time

Subenoy Chakraborty[1] and Tapan Kumar Ghosh[2]

Received June 9, 1998

We develop string cosmology in the presence of a Brans–Dicke (BD) scalar field coupled to Einstein gravity for higher dimensional space-time. Solutions are obtained for the equation of state for the p-string, and physical situations are discussed.

1. INTRODUCTION

It is a challenging problem to determine the exact physical situation at the very early stages of the formation of our universe. Among the various topological defects which occur in the early universe during the phase transition and before the creation of particles are strings, which have interesting cosmological consequences (Vilenkin, 1985).

The world sheets of strings are two-dimensional timelike surfaces (Chakraborty and Chakraborty, 1992). One can explain the present-day configurations of the universe by the large-scale network of strings in the early universe. Moreover, galaxy formation and the double quasar problem can be explained by density fluctuations of vacuum strings.

As the stress-energy of a string can be coupled to the gravitational field, it may be interesting to study the gravitational effects which arise from strings. In fact, the general relativistic treatment of strings was initiated by Letelier and Stachel and several authors have worked on it (Letelier, 1983).

However, it might be interesting to study string theory in the presence of a BD scalar field (Brans and Dicke, 1961). It is a modification of general relativity theory and is in accord with Mach's principle. The scalar field

[1] Department of Mathematics, Jadavpur University, Calcutta 700032, India.
[2] Natuk Vivekananda Bidyamandir, P.O. Natuk, Midnapore, West Bengal, India.

0020-7748/98/1000-240 9$15.00/0

which is introduced corresponds to a variable gravitational constant $\phi \sim G^{-1}$. The dimensionless coupling parameter ω between the scalar and tensor components of gravitation is constrained due to observational evidence. Further, both higher dimensional theory and string concepts are important at the early stages of the evolution of the universe. So it will be interesting to study string theory in a higher dimensional model.

2. THE BASIC EQUATIONS

We start with a five-dimensional homogeneous model with metric ansatz

$$dS^2 = -dt^2 + e^{\lambda}\, dr^2 + e^{\alpha}(d\theta^2 + \sin^2\theta\, d\phi^2) + e^{\mu}\, dy^2 \tag{2.1}$$

where $\lambda = \lambda(t)$, $\alpha = \alpha(t)$, and $\mu = \mu(t)$. Now we choose units such that $8\pi G = 1$ and use the comoving coordinate system; the velocity vector is taken to be

$$v^{\mu} = v_{\mu} = (1, 0, 0, 0, 0) \tag{2.2}$$

and the direction of the string x^{μ} is considered to be along the direction of $\partial/\partial r$, i.e., $x^{\mu} = (0, e^{-\lambda}, 0, 0, 0)$. Hence the five-velocity vector v^{μ} for the cloud of particles and the five-vector x^{μ}, the direction of the string, satisfy the standard relations (Banerjee *et al.*, 1990)

$$v_{\mu}\, v^{\mu} = -1 = -x_{\mu} x^{\mu} \qquad \text{and} \qquad v_{\mu} \cdot x^{\mu} = 0 \tag{2.3}$$

in $(-, +, +, +, +)$ signature for the space-time metric (2.1). Now, the energy-momentum tensor for a cloud of massive strings is (Letelier, 1983)

$$T_{\mu}^{\gamma} = \rho v_{\mu}\, v^{\gamma} - \Lambda x_{\mu} \cdot x^{\gamma} \tag{2.4}$$

where ρ, the rest energy for a cloud of strings, ρ_p, the particle energy density, and Λ, tension density of the string, are connected by the relation

$$\rho = \rho_p + \Lambda \tag{2.5}$$

Thus the surviving components of the Einstein equations coupled to the Brans–Dicke scalar field ϕ

$$R_{\mu\gamma} - \frac{1}{2} R \cdot g_{\mu\gamma} = \frac{T_{\mu\gamma}}{\phi} + \frac{\omega}{\phi^2}\left[(\partial_{\mu}^{\phi})(\partial_{\gamma}^{\phi}) - \frac{1}{2} g_{\mu\gamma}(\partial_{\rho}^{\phi})(\partial^{\rho}\phi) \right] + \frac{1}{\phi}(\phi_{;\mu;\gamma} - g_{\mu\gamma}\, \Box\phi) \tag{2.6}$$

and

$$\Box\phi = \frac{1}{2\omega + 3} T^{\mu}_{\mu} \tag{2.7}$$

are

$$\frac{\dot{\alpha}^2}{4} + \frac{\dot{\alpha}\dot{\lambda}}{2} + \frac{\dot{\alpha}\dot{\mu}}{2} + \frac{\dot{\lambda}\dot{\mu}}{4} + e^{-\alpha} = \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - \frac{1}{2}(\dot{\lambda} + 2\dot{\alpha} + \dot{\mu})\frac{\dot{\phi}}{\phi} + \frac{\rho}{\phi} \tag{2.8}$$

$$\ddot{\alpha} + \frac{3\dot{\alpha}^2}{4} + \frac{\ddot{\mu}}{2} + \frac{\dot{\mu}^2}{4} + \frac{\dot{\mu}\dot{\alpha}}{2} + e^{-\alpha}$$
$$= -\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - \frac{\ddot{\phi}}{\phi} - \frac{1}{2}(\dot{\mu} + 2\dot{\alpha})\frac{\dot{\phi}}{\phi} + \frac{\Lambda}{\phi} \tag{2.9}$$

$$\frac{\ddot{\alpha}}{2} + \frac{\dot{\alpha}^2}{4} + \frac{\ddot{\mu}}{2} + \frac{\dot{\mu}^2}{4} + \frac{\dot{\alpha}\dot{\mu}}{4} + \frac{\ddot{\lambda}}{2} + \frac{\dot{\alpha}\dot{\lambda}}{4} + \frac{\dot{\lambda}^2}{4} + \frac{\dot{\lambda}\dot{\mu}}{4} + e^{-\alpha}$$
$$= -\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - \frac{\ddot{\phi}}{\phi} - \frac{1}{2}(\dot{\lambda} + \dot{\alpha} + \dot{\mu})\frac{\dot{\phi}}{\phi} \tag{2.10}$$

$$\frac{\ddot{\lambda}}{2} + \frac{\dot{\lambda}^2}{4} + \ddot{\alpha} + \frac{3\dot{\alpha}^2}{4} + \frac{\dot{\lambda}\dot{\alpha}}{2} + e^{-\alpha} = -\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - \frac{\ddot{\phi}}{\phi} - \frac{1}{2}(\dot{\lambda} + 2\dot{\alpha})\frac{\dot{\phi}}{\phi} \tag{2.11}$$

and

$$\frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\frac{\dot{\lambda} + 2\dot{\alpha} + \dot{\mu}}{2} = \frac{1}{2\omega + 3}\frac{\rho + \Lambda}{\phi} \tag{2.12}$$

The overdot stands for d/dt.

The kinematic parameters which are of importance here, namely the proper volume R^4, expansion scalar θ, and shear scalar σ^2, have the following expressions for the above metric ansatz:

$$R^4 = \exp(\alpha + \lambda/2 + \mu/2)\sin\theta$$
$$\theta = 4\frac{\dot{R}}{R} = \dot{\alpha} + \frac{1}{2}(\dot{\lambda} + \dot{\mu}) \tag{2.13}$$
$$\sigma^2 = \frac{1}{6}[(\dot{\lambda} - \dot{\alpha})^2 + (\dot{\alpha} - \dot{\mu})^2 + (\dot{\mu} - \dot{\lambda})^2]$$

The Roychaudhuri equation can be obtained from the field equations and the above expressions for the kinematic parameters as

$$\dot{\theta} = -\frac{1}{3}\theta^2 - 2\sigma^2 - \omega\frac{\dot{\phi}^2}{\phi^2} + \frac{\dot{\phi}}{\phi}\left(\dot{\alpha} + \frac{\dot{\lambda} + \dot{\mu}}{2}\right)$$
$$- \frac{(4\rho + \omega\rho_p)}{\phi(2\omega + 3)} \tag{2.14}$$

The energy density for the coupled system ρ and ρ_p is restricted to be positive semidefinite by the energy conditions, while Λ is unrestricted in sign. However, one can interpret the field equations (2.8)–(2.11) with $\Lambda < 0$ as the presence of anisotropic fluid with pressure different from zero along the direction of the string (Banerjee *et al.*, 1990). Further, it is to be noted from the Roychaudhuri equation that the presence of the Brans–Dicke field coupled to the Einstein equations may halt the collapse that is present in the Einstein gravity.

3. SOLUTIONS TO THE FIELD EQUATIONS

To solve the field equations one notes that there are five field equations connecting six unknowns—three metric coefficients λ, α, and μ, the Brans–Dicke scalar ϕ, and two dynamical variables ρ and Λ. So usually one more relation connecting these variables is needed to solve the equations.

Let us assume one geometric relation, namely $\lambda = \alpha$, between two metric coefficients and another physical assumption, namely the equation of state $\rho = n\Lambda$ (n is a positive constant) for the p-string model. Then from the field equations we have the differential equation for λ,

$$A \frac{d^2\lambda}{dt^2} + B\left(\frac{d\lambda}{dt}\right)^2 + e^{-\lambda} = 0$$

where

$$A = \frac{3}{2} + n, \qquad B = \frac{3}{2}(1 + n) + n^2\left(\frac{\omega}{2} + 1\right)$$

The solution reads

$$\begin{aligned} e^{\lambda} &= \lambda_0(t - t_0)^2 = e^{\alpha} \\ e^{\mu} &= \mu_0(t - t_0) \\ \phi &= \phi_0(t - t_0)^{2k} \\ \Lambda &= \Lambda_0 t^{2k-2} \\ \rho &= n\Lambda \end{aligned}$$

where λ_0, μ_0, ϕ_0, Λ_0, and k are constants depending on n and ω, while t_0 is the constant of integration. So if we put $n = 1$ in the solution, then we have the geometric string model. Further, we note that with the evolution of the universe the scale factor corresponding to higher dimension also increases. Hence contraction of dimensions is not possible in this model.

For future work, it will be interesting to study the string model in general scalar-tensor theory.

REFERENCES

Banerjee, A., Sanyal, A. K., and Chakraborty, S. (1990). *Pramana,* **34**, 1.
Brans, C., and Dicke, R. (1961). *Physical Review,* **124**, 925.
Chakraborty, S., and Chakraborty, A. K. (1992). *Journal of Mathematical Physics,* **33**, 2336.
Letelier, P. S. (1983). *Physical Review D,* **28**, 2414.
Vilenkin, A. (1985). *Physics Reports,* **121**, 263.